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THE EFFECT OF ROUNDING THE LEADING EDGES ON THE CHARACTERISTICS OF SEPARATED FLOW PAST DELTA WINGS OF LOW ASPECT RATIO

[VLIYANIE ZATUPLENIYA PEREDNIKH KROMOK NA KHARAKTERISTIKI OTRYVNOGO OBTEKANIYA TREUGOL'NYKH KRYL'EV MALOGO UDLINENIYA]

by

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AUTHOR'S SUMMARY

The separated flow of an ideal fluid around delta wings of low aspect ratio with rounded leading edges is calculated by an approximation to the theory of slender bodies. The numerical method employed is based upon a mathematical model of inviscid separation from a smooth surface. Results of calculations are given for symmetrical flow around wings whose cross-sections are ellipses. Particular attention is paid to the investigation of the influence of the position of the line of separation in the vicinity of the rounded edge of the wing on the configuration of the vortex sheet and to the overall aerodynamic characteristics of the wing.

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Experimental results show that the characteristics of separated flow around delta wings of low aspect ratio depend substantially upon the form of the leading edges[1]. Existing methods of calculation of such flows[2,3] within the constraints of an ideal fluid are based upon the assumption of sharp edges on the wing, even when the thickness of the wing is taken into consideration[4]. Although such a constraint on the shape of the wings does allow a substantial simplification of the task (complying with the Joukowski hypothesis for sharp wings ensures a unique solution), the area of application of these methods is restricted. In instances when these methods are applied to the calculation of flow around delta wings with rounded leading edges, they do not achieve solutions of the required accuracy. Rounding of the wings gives rise to a finite range of angles of attack in which the flow is generally unseparated. At large angles of attack, separated flow occurs, but the supplementary increment in lifting force is significantly less than in the case of flow around the same wings with sharp edges[1].

For the calculation of the characteristics of separated flow around wings of low aspect ratio, for which the radii of rounding of the edges is much greater than the thickness of the boundary layers, a method may be used, based upon a model of inviscid separated flow around bodies with a smooth surface. This model is characterised by the non-unique solutions obtained. In particular, there may occur a single-parameter set of solutions with the position of the lines of separation as an independent parameter, as, for example, in the problem of symmetrical separated flow around a thin circular cone[5].

The aim of this article is the construction of single-parameter sets of solutions to the problem of symmetrical separated flow around delta wings of low aspect ratio, whose cross-sections are ellipses of small relative thickness; also an analysis on the basis of these solutions of the influence of the position of the line of separation, the rounding of the edges and the thickness of the wing on the vortex system. The aim is also to obtain the overall aerodynamic characteristics.

The question of the extraction of the unique solution from the corresponding single-parameter set of solutions goes beyond the limits of the present investigation. The answer to this may be found either experimentally (the determination of the position of the line of separation in each particular case), or theoretically by a special investigation of both the regions of flow in which viscous forces play a substantial part, and the application of the corresponding procedures for bringing together the solutions in these regions[6].

1 Stating the problem. Assume a right-angled Cartesian system of co-ordinates with its origin at the wing apex. Let the axis Oz be directed along the axis of symmetry of the wing, the axis Ox along the span and the axis Oy perpendicular to the axes Oz and Ox. The wing is subject to a flow of gas around it and at an angle of attack α with a velocity vector ω_∞ at infinity, and lying in the (y,z) plane (Fig 1).

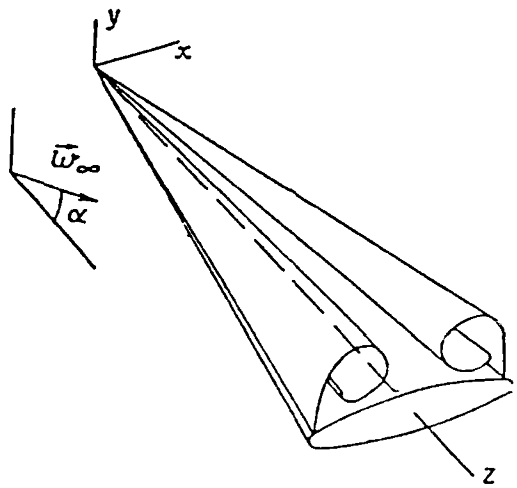

Fig 1

In accordance with the results of the Ward-Jones theory of slender bodies[7], the velocity potential $\phi(x,y,z)$ at distances of the order of the typical dimension of the cross-section may be expressed in the following form:

$$\phi(x,y,z) = \omega_\infty\left[\left(1 - \frac{\alpha^2}{2}\right)z + g_0(z)\right] + \phi_1(x,y,z) \quad ,$$

where with $M_\infty < 1$

$$2\pi g_0(z) = S'(z)\ \ln\frac{\sqrt{1 - M_\infty^2}}{2} - \frac{1}{2}\int_0^1 S''(\mu)\ \mathrm{sign}(\mu - z)\ \ln|\mu - z|d\mu$$

and with $M_\infty > 1$, $S'(0) = 0$

$$2\pi g_0(z) = S'(z)\ \ln\frac{\sqrt{M_\infty^2 - 1}}{2} - \int_0^z S''(\mu)\ \ln(z-\mu)d\mu \quad .$$

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Here M_∞ is the quantity M of the airstream, $S(z)$ the area of the wing cross-section, $0 \leqslant z \leqslant 1$. The function ϕ_1 is harmonic in the variables $x, y(\phi''_{1xx} + \phi''_{1yy} = 0)$ and for $x^2 + y^2 \to \infty$ may be expressed in the form:

$$\frac{\phi_1}{\omega_\infty} \approx \alpha y + \frac{S'(z)}{2\pi} \ln\sqrt{x^2 + y^2} \quad .$$

Also, the expression $\omega_\infty\left(1 - \frac{\alpha^2}{2}\right)z + \phi_1(x,y,z)$ satisfies the condition for absence of flow through the surface of the wing and the boundary conditions on the vortex sheets.

After introducing the time-variable $t = z/\omega_\infty$, the three-dimensional stationary problem of the separated flow around the wing under examination reverts to the two-dimensional unsteady (similarity) problem of the separated flow of an incompressible fluid which expands uniformly in time around the elliptical contour L_0 (Fig 2a).

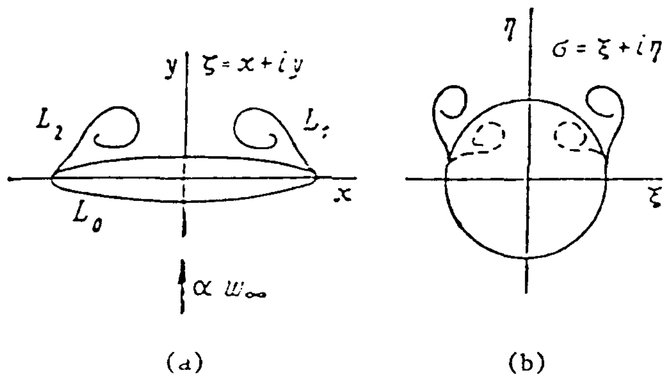

Fig 2

For a solution of the above we use the methods of the theory of functions of a complex variable. In the plane of the cross-section of the wing we introduce the complex variable $\zeta = x + iy$ and examine the flow picture at an arbitrary fixed moment in time. Let the configuration and intensity of the vortex sheets L_1 and L_2 be given in the parametric form

$$\zeta_{1,2} = \zeta_{1,2}(\Gamma, t) \quad , \qquad 0 \leqslant |\Gamma| \leqslant |\Gamma^*_{1,2}| \quad ,$$

where Γ is the circulation of the section of the vortex sheet, measured from the core (the actual parameter). Their magnitudes or signs at the points of separation of the flow are denoted both here and in the following text by an asterisk and subscript.

Let us formulate the following problem: it is required to find the complex velocity of flow $V(\zeta,t)$ which is regular outside the contour $L = L_0 + L_1 + L_2$, and possessing the specified discontinuity on the vortex sheets L_1 and L_2

$$[V_+(\zeta,t) - V_-(\zeta,t)]_{\zeta\epsilon L_1 L_2} = \gamma_{1,2}(\zeta,t) = 1\Big/\frac{\partial\zeta_{1,2}}{\partial\Gamma} ,$$

where "+" and "-" indicate the signs of V from the two sides of the sheet, γ represents the linear density of the circulation; the complex velocity is also to satisfy the boundary conditions for the absence of flow on the contour L_0 and the attenuation of perturbations at infinity

$$V(\zeta,t)_{|\zeta|\to\infty} \to - i\alpha\omega_\infty .$$

The sought-after complex velocity satisfying all the stated conditions may be readily obtained if the corresponding complex potential of the flow is known. To find the latter we use the conformal mapping of the outside of the ellipse with semi-major axes $a(t)$ and $b(t)$ in the plane of the variable ζ onto the outside of the circle of radius $R(t)$ in the plane of the complex variable $\sigma = \xi + i\eta$ (Fig 2b)

$$\sigma = \tfrac{1}{2}[\zeta + (\zeta^2 - d^2)^{\frac{1}{2}}] ,$$

where $d = a\sqrt{1 - \varepsilon^2}$, $R = a(1 + \varepsilon)/2$, $\varepsilon = b/a$, the relative thickness of the wing. The inverse mapping has the form

$$\zeta = \sigma + \frac{d^2}{4\sigma} .$$

In the particular case of a symmetrical scheme of separated flow we have for the complex potential $W(\sigma,t)$ of the flow under investigation

$$W = \varepsilon a \frac{da}{dt} \ln \sigma - i\alpha\omega_\infty\left(\sigma - \frac{R^2}{\sigma}\right) - \frac{i}{2\pi}\int_{L_1} \ln \frac{(\sigma - \sigma_0)\left(\sigma + \frac{R^2}{\sigma_0}\right)}{(\sigma + \sigma_0)\left(\sigma - \frac{R^2}{\bar{\sigma}_0}\right)} d\Gamma_0 . \quad (1.1)$$

Both here and in the following text, the variable with the bar indicates the complex conjugate of the quantity. The link between the complex potential and the required flow velocity in the plane of the variable ζ is expressed in the usual manner

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$$V(\zeta,t) = \frac{dW}{d\sigma}\left(\frac{d\sigma}{d\zeta}\right) . \tag{1.2}$$

As soon as the complex flow velocity for the arbitrary fixed moment in time is obtained, the configuration and intensity of the vortex sheets at subsequent moments of time may be obtained as a result of Cauchy's solution for singular integro-differential equations[8]

$$\frac{\partial\bar{\zeta}_{1,2}}{\partial t} = V[\zeta_{1,2}(\Gamma,t),t] , \qquad 0 \leqslant |\Gamma| \leqslant |\Gamma^*_{1,2}|, \tag{1.3}$$

which are able to fulfil the conditions of continuity of pressure and normal component of flow velocity on the vortex sheets.

2 Method of solution. The Cauchy solution as expressed above (1.1) to (1.3) is applied to the vortex sheets at a given fixed moment of time in the flow. To obtain the sections of the vortex sheets leaving the surface of the wing and entering the stream in the interval of time from the given value to the next, it was proposed[9] to use the equation for the density of the circulation at the point of separation γ^* and the expression for the velocity of shift of the point of separation v^* along the smooth contour of a body under the condition $\rho = r$, where ρ is the radius of curvature of the vortex sheet, and r is the radius of curvature of the surface of the body.

$$\left(\frac{d\gamma}{dt}\right)^* = -\gamma^*\left(\frac{\partial v_{s+}}{\partial s} + \frac{v_n}{\rho}\right) , \qquad v^* = (v_{s-})^* , \tag{2.1}$$

Here v_{s+}, v_{s-} are the tangential components of velocity from the outer and inner sides of the sheet respectively, v_n is the normal component of velocity on the sheet, and s is the length of the arc.

As shown[5,10], the integration of the equation (2.1) with $\rho = r$ in common with equations (1.1) to (1.3) leads to the achievement of a unique solution satisfying the Brillouin-Villat conditions: at the points of separation there is zero gradient of pressure and the curvature of the sheet equals the curvature of the body. Analysis of the asymptote of the flow in the vicinity of the point of separation reveals the indeterminacy of the expression in curved brackets in equation (2.1), arising in the case when solutions are sought which do not satisfy the Brillouin-Villat conditions. As is known[6], these solutions are characterised by an infinite curvature of the vortex sheet at the point of separation, and consequently by an infinite pressure gradient at this point.

The discovery of the indeterminacy in (2.1) leads to an equation in which an arbitrary function occurs, and in the case of self-modelling flow, an arbitrary constant[10], indeterminable within the constraints of an ideal fluid. In determining the significance of this constant, solutions may be obtained for similarity problems of separated flow with a different position of the separation points. Such an implicit operation for obtaining the positions of separation was used[5] to obtain the single-parameter set of solutions for the inviscid symmetrical separated flow around a thin circular cone.

In the present work, an explicit programme for the separation points on an elliptical contour was used. Even if such an approach leads to a disturbance in the equations of motion in the process of determining the solution, it is justified for self-modelling problems, characterised by the conditions

$$\left(\frac{d\gamma}{dt}\right)^* = 0 \ , \qquad v^* = 0 \ , \qquad \gamma^* = (v_{s+})^* \ .$$

The process of determining the solutions in this case is accelerated, insofar as there is an absence of the oscillatory movement of the points of separation along the contour of the body, which takes place while obtaining a solution with an implicit programme of the position of separation.

The impossibility of obtaining a solution precisely satisfying the Brillouin-Villat conditions is in some way a shortcoming of the approach under investigation. However, the position of the point of separation corresponding to the specified condition may be determined with sufficient accuracy on account of the absence of convergence in the process of determining the earlier (upstream) positions of the separation points.

In other respects the numerical method used corresponds to the one quoted[5], including: the discretization of the vortex sheet; the integration of the system of equations of motion of the discrete vortices according to a regularised scheme having a first order of accuracy; the use of a model of the core of the sheet (vortex and cut); the improved approximation of velocity along the normal to the vortex sheet, at intervals less than the steps of the discretization.

3 Results of the calculations. In view of the large number of parameters in the problem under investigation, calculations were provided at fixed values of the relative angle of attack $\bar{\alpha} = \alpha/\chi = 1.5$, where χ is the half-angle at the apex of the wing in the plane (x,z) for three wings with relative thicknesses $\varepsilon = 1/7; 1/10; 1/14$. In accordance with the experimental data[11], with a fixed

value of the relative angle of attack the lines of separation on elliptical cones with a diminished relative thickness are directed towards a generator passing through the tips of the ellipses, and ending on the upper side of their surface.

Arising from this, the vicinity of the ends of the large semi-major axes of the ellipses was chosen to represent the range of variation in the position of the line of separation. Let us agree in future to refer to this as the vicinity of the rounded edge, and the geometrical position of the tips of the ellipses, at which the curvature is a maximum, as the edge itself.

Let us introduce into the section $z = \text{const}$ a curvilinear system of co-ordinates x_1, y_1. The abscissa x_1 is referred to the edge along the contour of the ellipse, and the ordinate y_1 to the outer normal to the contour. We will designate the co-ordinates of the point of separation in this system of co-ordinates by $x_1^*, y_1^* \equiv 0$. For the configuration of the vortex sheet in the vicinity of the point of separation, the approximation is justifiably made

$$y_1 \approx c_0(x_1^* - x_1)^{3/2} , \qquad x_1 \leqslant x_1^* < x_1^{**} ,$$

and correspondingly in the limiting case of separated flows satisfying the *Brillouin-Villat* conditions,

$$y_1 \approx c_1(x_1^{**} - x_1)^{5/2}, \qquad x_1 \leqslant x_1^{**} ,$$

where x_1^{**}, $y_1^{**} \equiv 0$ are the co-ordinates of the point of separation in this limiting case.

The coefficients c_0, c_1 represent functions of $\bar{\alpha}$, ε, and the coefficient c_0 also represents a function of $(x_1^{**} - x_1^*)$.

At the right of Fig 3 the vicinities of the rounded edges are shown together with those regions under investigation of the positions of the lines of separation on all three wings. The upper boundaries of the regions (see Fig 3) are arbitrarily chosen.

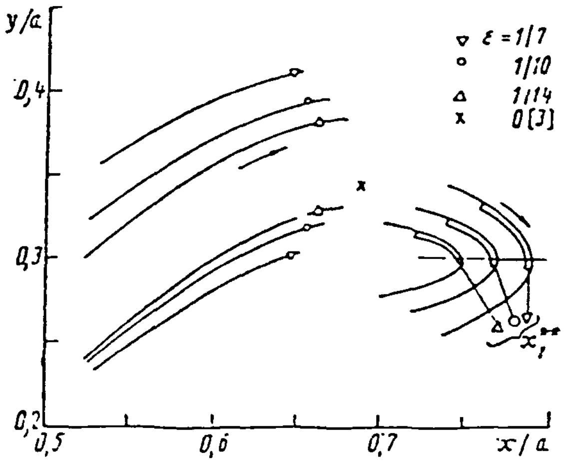

Fig 3

With regard to the lower boundaries of the region under investigation it is possible almost to take them beyond the limits of existence of solutions (they correspond closely to solutions fulfilling the Brillouin-Villat conditions at the point of separation). As already noted above, the accurate achievement of solutions satisfying these conditions is not possible by the method employed in this work of directly programming the positions of the separation points. The behaviour of the lower boundaries of the regions of the position of the line of separation under investigation is consistent with the tendency of x_1^{**} towards the edge of the wing with a decrease in relative thickness ε and consequently in the radius of the rounded edge $(\sim\varepsilon^2)$.

At the left of Fig 3, the change of position of the cores of the vortex sheets for the ranges of positions of the line of separation under investigation is shown in x,y co-ordinates (see Fig 2a) in continuous lines to the same scale. The arrow indicates the direction of the change in position of the cores of the sheets, this direction corresponding to the change in the line of separation from the highest to the lowest boundaries of the ranges under investigation. The symbols on the curves indicate the position of the cores of the vortex sheets at separation from the edges $(x_1^* = 0)$. The small displacement of the separation points in the vicinity of the rounded edges (especially on the upper surface of the wing) leads to a substantial displacement of the cores of the vortex sheets, the larger, the smaller the value of ε . For comparison, in the same place the position is shown of the core of the vortex sheet for a triangular lamina with sharp edges[3] for the case when the Joukowski hypothesis is fulfilled at these edges. The form of distribution of the curves for the various values of ε , also the analysis of the results[4] concerning the influence of small thickness on

the characteristics of separated flow around delta wings with sharp leading edges allow one to propose the existence of a simple mechanism for the displacement of the vortex spiral along the y ordinate on the upper surface of the wing. If we subtract from the ordinate of the position of the core of the vortex sheet the ordinate of the upper surface of the wing with the same abscissa, then we obtain the curves shown at the lower left of Fig 3. The trend of the curves towards the 'plane' (y = 0) of the upper surface of the wing is consistent with the tendency of those solutions obtained for the position of the cores of the vortex sheets to approach that for a plane triangular lamina with sharp leading edges, with reduced relative thickness and under the condition of maximum early separation from the windward surface of the wing (meeting the Brillouin-Villat conditions at the separation points).

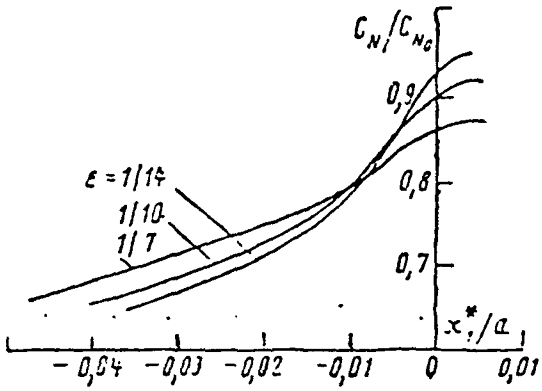

Fig 4

In Fig 4 is shown the dependency of the coefficient of normal force C_N of the wings under examination on the co-ordinate of the separation point in the vicinity of the rounded edge. The coefficient of normal force is related to the coefficient of normal force C_{N0} of a triangular lamina with sharp edges: $C_{N0}/\chi^2 = 18.82$ with $\bar{\alpha} = 1.5$ (see Ref 3). The maximum value of the coefficient of normal force, as in the case of separated flow around a circular cone[5], occurs when the Brillouin-Villat conditions are met at the points of separation. With a reduction in the relative thickness of the wing ε , an increment is observed in this maximum value and its tendency towards the coefficient of normal force for a triangular lamina with sharp edges.

Fig 5 illustrates the configurations of the vortex sheets for the extreme points of the investigated ranges of positions of the line of separation on a wing having relative thickness $\varepsilon = 1/10$.

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The investigations conducted show that even an insignificant displacement of the position of the lines of separation in the vicinity of rounded leading edges on the upper surface of the wing (in calculations, the displacement did not exceed 3% of the half-span) leads to a substantial reduction in the supplementary normal force brought about by the separated flow. For example, for a wing with relative thickness $\varepsilon = 1/10$, the supplementary normal force is decreased by 62%.

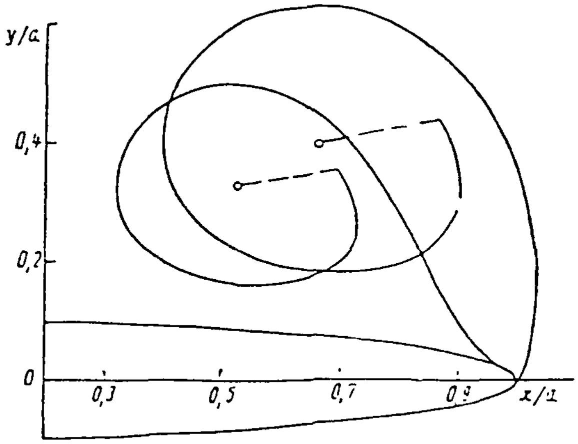

Fig 5

The above discussion supports the possibility of setting-up a numerical method for the calculation of the characteristics of separated flow around wings of small aspect ratio, having thickness and rounded leading edges, within the constraints of a model of an ideal fluid. For its effective use, it is necessary to have means of determining with the highest possible accuracy (experimentally or theoretically) the position of the lines of separation in the vicinity of the leading edges under given specific conditions of flow.

REFERENCES

No.	Author	Title, etc
1	G.E. Bartlett R.G. Vidal	Experimental investigation of influence of edge shape on the aerodynamic characteristics of low aspect ratio wings at low speeds. *JAS*, 22, 8, 1955
2	S.M. Byelotserkovski M.I. Nisht	Otryvnoe i bezotryvnoe obtekanie tonkikh kryl'ev ideal'noi zhidkost'yu. [Separated and unseparated flow of an ideal fluid around thin wings] M., 'Nauka', 1978
3	J.H.B. Smith	Improved calculations of leading-edge separation from slender, thin, delta wings. *Proc. Roy. Soc.*, A306, 1968
4	G.G. Sudakov	Raschet otryvhykh techenii okolo konicheskikh kryl'ev maloi tolshchiny. [Calculation of separated flows around conical wings of small thickness] *Uchenye zapiski TsAGI*, 5, 6, 1974
5	S.B. Zakharov	Raschët nevyazkogo otryvnogo obtekaniya tonkogo krugovogo konusa na bol'shikh uglakh ataki. [Calculation of inviscid separated flow around a thin circular cone at large angles of attack] *Uchenye zapiski TsAGI*, 7, 6, 1976
6	V.V. Sychev	O laminarnom otryve. [Laminar separation] *Isv. AN USSR, MZhG*, No.3, 1972
7	G.N. Ward	Linearised theory of steady high-speed flow. Cambridge University Press, 1955
8	A.A. Nikol'ski	O 'vtoroi' forme dvizheniya ideal'noi zhidkosti okolo obtekaemogo tela (issledovanie otryvnykh vikhrevykh potokov. ['Second' form of movement of an ideal fluid around a body (investigation of separated vortex flows)] *DAN USSR*, 116, 2, 1957

REFERENCES (concluded)

No.	Author	Title, etc
9	K.P. Il'ichev S.N. Postolovski	Raschët nestatsionarnogo otryvnogo obtekaniya tel ploskim potokom nevyazkoi zhidkosti. [Calculation of non-stationary separated flow around bodies by a plane stream of inviscid fluid] *Isv. AN USSR, MZhG,* No.2, 1972
10	S.K. Betyaev S.B. Zakharov V.F. Molchanov G.G. Sudakov	Nekotorye zadachi teorii otryvnykh techenii ideal'noi zhidkosti i gaza. [Certain problems of the theory of separated flow of an ideal fluid and gas] Trudy odinnadtsatykh chtenii, posvyashchennykh razrabotke nauchnogo nalediya i razvitiyu idei K.E. Tsiolkovskogo. [Transactions of the eleventh series of lectures dedicated to the development of the scientific legacy and ideas of K.E. Tsiolkovski] *M., IIET AN USSR,* 1978
11	L.H. Jorgensen	Elliptic cones alone and with wings at supersonic speeds. NACA Report 1376 (1958)